

1.

In United States a date such as July 4, 2024, is often written 7/4/24, but in other countries the month is given second and the same date is written 4/7/24. If you don't know which system is being used, how many dates in a year are ambiguous in this two-slash notation?

For example, 01/05/24 is ambiguous, as it could mean January 5, 2024 (in the United States) or May 1, 2024 (in many other countries). However, 02/02/24 is not ambiguous, as both formats interpret it as February 2, 2024.

Answer:

132

Solution:

Each month has 11 ambiguous dates, making 132 in all.

For example, in January, the following dates are ambiguous.

01/02/24

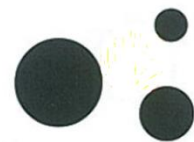
01/03/24

...

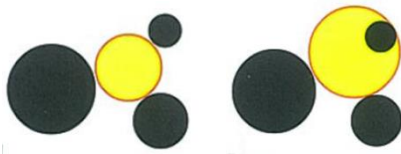
01/12/24

2.

Given three circles in the plane as shown below, in how many different ways can a fourth circle be added so that they are all tangent (touching in a point at the circumference of the fourth added circle)?



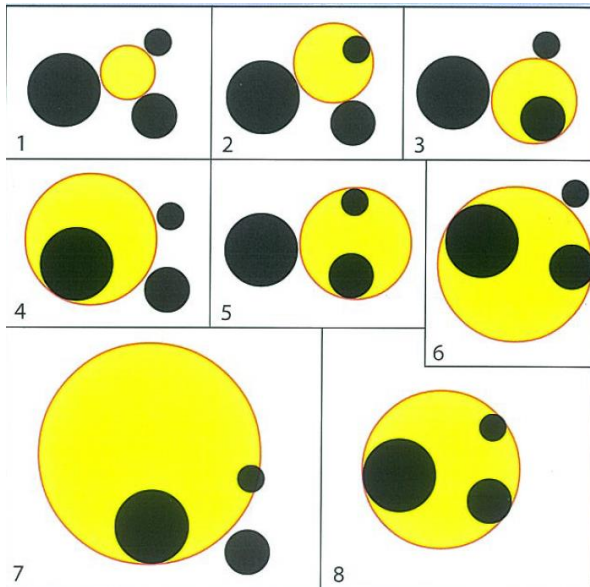
Hint: Two examples are as follows.



Answer: 8

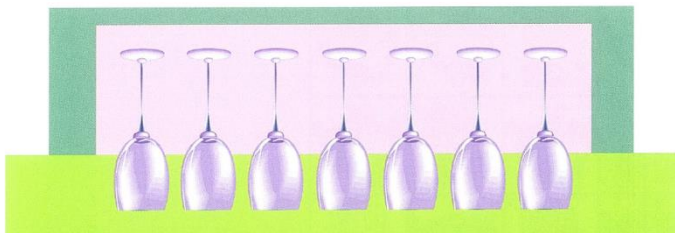
Solution:

There are only eight different possibilities, as shown below.



3.

Seven glasses face down, as shown.



The objective is to turn all seven glasses face up by inverting three glasses (not necessarily three consecutive glasses) in each move.

Note: In each move, you need to invert exactly three glasses (not necessarily three consecutive glasses). For example, if you invert the three glasses on the left, you will get the following.



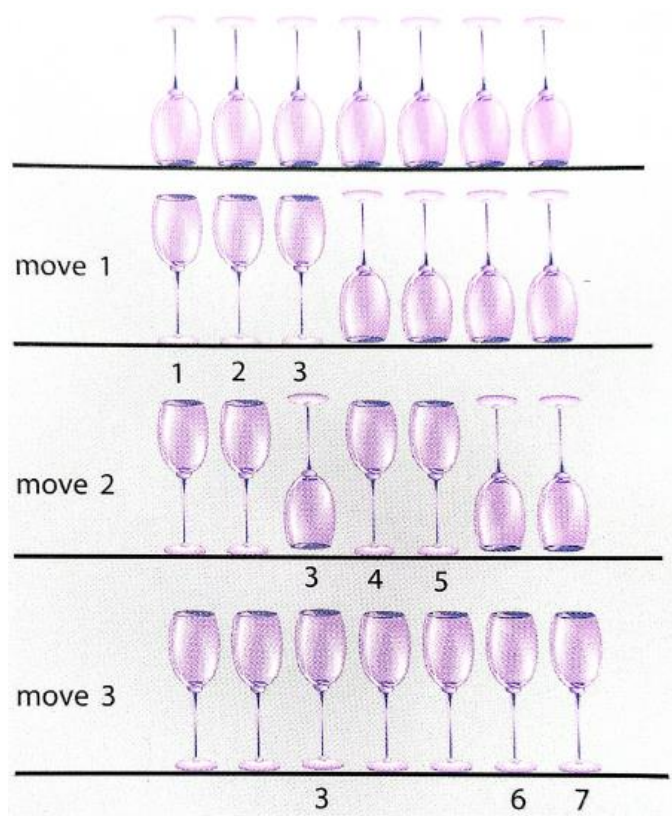
Please note the three glasses that you invert are not necessarily three consecutive glasses.

In at least how many moves can it be done?

Answer: 3

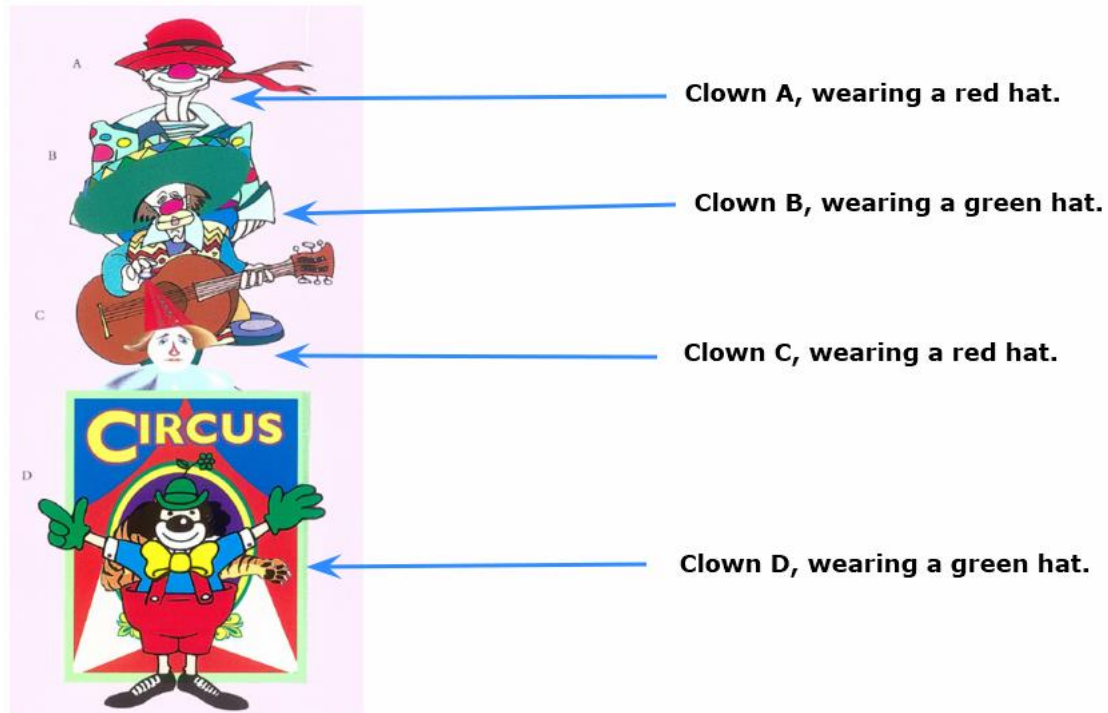
Solution:

Three moves are necessary, as shown.



4.

Four clowns are wearing two red and two green hats. They know that there are two hats of each color, but none of them knows the color of his own hat, and they are not allowed to turn and look behind them.



Which of them will be the first to shout out the color of his hat?

- (a) Clown A
- (b) Clown B
- (c) Clown C
- (d) Clown D

Note: Clown D cannot be seen by the others, since he is obstructed by the circus poster. Therefore, Clown C cannot see any hat, Clown B can see only one red hat, and Clown A can see only one green hat and one red hat.

Answer: (b)

Solution:

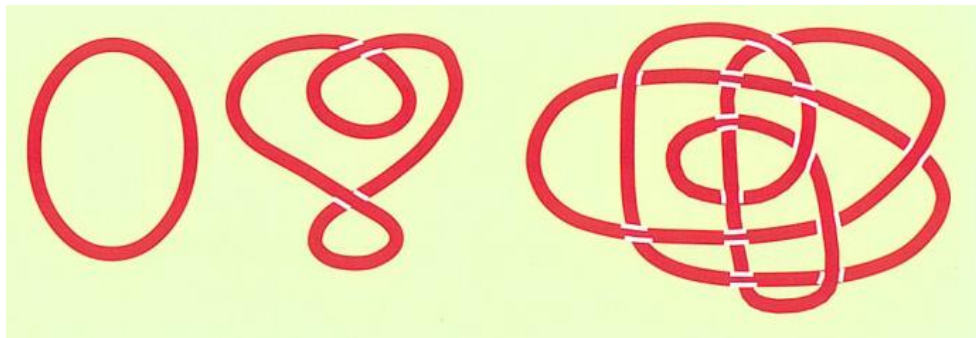
Clown B.

If clown A saw two red or two green hats he would know the color of his hat (as well as the color of clown D's hat). But what he sees is one of each color, which doesn't offer him any clues.

Hearing that clown A is silent, clown B can deduce that the color of his hat must be the opposite color of the hat in front of him.

5.

Two knots are equivalent if one can be transformed into the other. A special case is to recognize the unknot from a real knot.



The figures on the left and in the middle show an unknot and its equivalent. Is the figure on the right an unknot?

Answer: Yes

Solution:

The right figure is also an unknot.