Instructions

This test is to assess your **ability to explain mathematical reasoning clearly.**

Example 1:

Problem:
How many integers between 1 and 1000 (inclusive) are divisible by 3 or 5 or 7?

Solution:

We are asked to count the number of integers from 1 to 1000 that are divisible by **at least one** of 3, 5, or 7.

This is a classic application of the **inclusion-exclusion principle.**

Let:

- A = numbers divisible by 3
- B = numbers divisible by 5
- C = numbers divisible by 7

We want:

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

Let's compute each term:

Step 1: Individual Counts

- $|A| = \left\lfloor \frac{1000}{3} \right\rfloor = 333$
- $|B| = \left\lfloor \frac{1000}{5} \right\rfloor = 200$
- $|C| = \left\lfloor \frac{1000}{7} \right\rfloor = 142$

Step 2: Pairwise Intersections

- $|A \cap B| = \left\lfloor \frac{1000}{15} \right\rfloor = 66$
- $|A \cap C| = \left\lfloor \frac{1000}{21} \right\rfloor = 47$
- $|B \cap C| = \left\lfloor \frac{1000}{35} \right\rfloor = 28$

Step 3: Triple Intersection

- $|A \cap B \cap C| = \left\lfloor \frac{1000}{105} \right\rfloor = 9$

Step 4: Inclusion-Exclusion Total

$$|A \cup B \cup C| = 333 + 200 + 142 - 66 - 47 - 28 + 9 =$$

$$= 675 - 141 + 9 = 543$$

Answer:

$$\boxed{\text{543 integers between 1 and 1000 are divisible by 3, 5, or 7.}}$$

Why this is a strong write-up:

1. It uses **clear notation** and organized step-by-step reasoning.
2. Demonstrates knowledge of the **inclusion-exclusion principle**.
3. Performs accurate calculations with **modular thinking**.
4. Communicates effectively without over-complication.

Example 2:

Problem:
Prove that in any group of 13 people, at least two of them will have their birthdays in the same month.

Solution:

There are **12 months** in a year. Imagine we have **13 people**, and we want to assign each person's birthday month to one of these 12 months.

Now apply the **Pigeonhole Principle**, which states:

> If you place more items (pigeons) into fewer categories (pigeonholes) than there are items, at least one category must contain more than one item.

In this case:

- **Pigeons** = people = 13
- **Pigeonholes** = months = 12

By the Pigeonhole Principle, if we assign 13 people to 12 months, at least **one month must contain the birthdays of at least two people.**

Conclusion:

$$\boxed{\text{At least two people will have their birthdays in the same month.}}$$

Why this is a strong write-up:

1. It clearly defines the elements involved.
2. It correctly identifies how the Pigeonhole Principle applies.
3. It concludes with a general and understandable result.
4. It uses precise language and logical structure.

Example 3:

Problem:
There are 100 people in a room. Each person shakes hands with exactly 10 others. Show that the total number of handshakes is divisible by 5.

Solution:

1. **Count handshakes in two ways**
   - Every person shakes hands with **10 people**.
   - There are **100 people**.
   - So if you count all the "handshakes from a person's point of view," you get:

$$100 \times 10 = 1000$$

2. **Correct the double counting**
   - But notice: when person A shakes hands with person B, **both A and B count the same handshake**.
   - So we have counted every handshake **twice**.
3. **Divide by 2**
   - The actual number of handshakes is

$$\frac{1000}{2} = 500$$

4. **Check divisibility by 5**
   - $500 \div 5 = 100$, which is a whole number.
   - So 500 is divisible by 5.

✅ Therefore, the total number of handshakes is **500**, and it is divisible by 5.

Why this is a strong write-up:

1. **Clear Step-by-Step Logic**
   - It doesn't jump to the answer immediately.
   - It shows *how* to count the handshakes and *why* we divide by 2.
2. **Uses Easy Numbers**
   - Starts with $100 \times 10 = 1000$, a simple multiplication.
   - Then divides by 2, which is easy to follow.
3. **Checks the Goal Clearly**
   - It not only finds the total (500) but also explicitly shows why 500 is divisible by 5.
4. **Logical Flow**

- **Count → Notice double counting → Correct → Conclude.**
- Each step follows naturally from the one before.

In short: It's a strong write-up because it is **clear, simple, and convincing** — the reader can follow the reasoning, which proves the result rigorously.

Important:

Please note that you don't need to write as eloquently or elegantly as the three examples above. What matters is clarity. As long as your reasoning is clear and you present it clearly, that will be sufficient.

This test is to assess your **ability to explain mathematical reasoning clearly.**

What we look for:
1. Mathematical clarity and rigor
2. Communication skills
3. Original reasoning (not just memorized tricks)
4. Precision in logic, definitions, and structure

What Makes a Strong Write-Up?
1. Clarity: Use clear, concise language with good structure.
2. Logical flow: Start with what's known, proceed step-by-step to the conclusion.
3. Definitions: Define your variables and state any theorems you use.
4. Creativity: Some problems benefit from a clever insight — don't be afraid to be original!